

## Secondary Course Description

| COVER PAGE   |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
|--|--|-----------------|-------|-----------|--|-------|---|-----------------|-------|-------|---|-----------------|-------|
| 1. Course Title: Integrated Mathematics II   | 2. Transcript Title / Abbreviation: Int. Math II   |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| 3. Transcript Course Code / Number (Office Use Only):  | 4. School: Davis Senior High School, Harper Junior High School, Holmes Junior High School, Emerson Junior High School, Da Vinci Junior and High School, Davis School for Independent Study, King High School             |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| 5. District: Davis Joint Unified School District   | 6. Length of Course: One Year – Fall/Spring  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| 7. School / District Web Site: <a href="http://www.djUSD.net">http://www.djUSD.net</a>   | <b>8. School Contact</b><br>Name: Stephanie Gregson<br>Title/Position: Director, Curriculum & Instruction<br>Phone: 530/757-5300 Ext.: 150<br>Fax:<br>E-mail: <a href="mailto:sgregson@djUSD.net">sgregson@djUSD.net</a> |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>9. Subject Area:</b><br><div style="display: flex; flex-wrap: wrap;"> <div style="width: 33%;"><input type="checkbox"/> History/Social Science</div> <div style="width: 33%;"><input type="checkbox"/> English</div> <div style="width: 33%;"><input checked="" type="checkbox"/> Mathematics</div> <div style="width: 33%;"><input type="checkbox"/> Science</div> <div style="width: 33%;"><input type="checkbox"/> Language other than English</div> <div style="width: 33%;"><input type="checkbox"/> Visual &amp; Performing Arts</div> <div style="width: 33%;"><input type="checkbox"/> DJUSD Graduation Elective</div> <div style="width: 33%;"><input type="checkbox"/> College Prep Elective (will seek UC/CSU approval)</div> </div> |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| 10. Grade Level(s): 9-12   | 11. Seeking "Honors" Distinction? <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>12. Credit Value:</b><br><input type="checkbox"/> 0.5 (half year or semester equivalent) <input checked="" type="checkbox"/> 1.0 (one year equivalent)<br><input type="checkbox"/> 2.0 (two year equivalent) <input type="checkbox"/> Other: _____  |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>13. Was this course previously approved by UC?</b> <input type="checkbox"/> Yes <input checked="" type="checkbox"/> No<br>If so, in what year? _____ Under what course title? _____   |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>14. Pre-Requisites:</b> Integrated Mathematics I<br><b>Co-Requisites:</b> None  |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>15. Preliminary Approval - Secondary Site Principal Signature (Must be signed before proceeding to Step 16):</b><br>_____   |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>16. Date Course Proposal with Preliminary Approval (Step 15) sent to Associate Superintendent, Educational Services:</b> _____  |  |                 |       |           |  |       |   |                 |       |       |   |                 |       |
| <b>17. Review &amp; Approval:</b><br><table style="width: 100%; border: none;"> <tr> <td style="width: 15%;">Date</td> <td style="width: 40%;"></td> <td style="width: 15%; text-align: right;">Signature</td> <td style="width: 30%;"></td> </tr> <tr> <td>_____</td> <td>Site Curriculum and Instruction Leadership Team</td> <td style="text-align: right;">Signature/Title</td> <td>_____</td> </tr> <tr> <td>_____</td> <td>Secondary Department Articulation/Collaboration</td> <td style="text-align: right;">Signature/Title</td> <td>_____</td> </tr> </table> Secondary Principals Signatures: _____<br>Date: _____  |  | Date            |       | Signature |  | _____ | Site Curriculum and Instruction Leadership Team | Signature/Title | _____ | _____ | Secondary Department Articulation/Collaboration | Signature/Title | _____ |
| Date   |  | Signature       |       |           |  |       |   |                 |       |       |   |                 |       |
| _____  | Site Curriculum and Instruction Leadership Team  | Signature/Title | _____ |           |  |       |   |                 |       |       |   |                 |       |
| _____  | Secondary Department Articulation/Collaboration  | Signature/Title | _____ |           |  |       |   |                 |       |       |   |                 |       |

**BACKGROUND INFORMATION**

In 2010 the California State Department of Education adopted the Common Core State Standards for Mathematics. The goal of the Common Core State Standards for Mathematics is for students to be college and career ready upon graduation from high school and to assist students in becoming competitive in a global economy. Therefore, the Common Core State Standards for Mathematics provide not only for rigorous curriculum and instruction, but also conceptual understanding, procedural skill and fluency and the ability to apply mathematics. Students will develop the skills to be able to problem-solve creatively and not be satisfied by just arriving at an answer, thus meeting the challenges of the 21st century.

These new Standards for Mathematics have been developed to provide students with the knowledge, skills, and understanding in mathematics to be college and career ready when they complete high school. They are internationally benchmarked and assist students in their preparation for enrollment at a public or private university.

The Common Core State Standards for Mathematics include two types of standards:

1. Eight Standards for Mathematical Practice that are the same in each grade level and high school mathematics course.
2. Mathematical Content Standards that are organized into high school courses.

“Together these standards address both ‘habits of mind’ that students should develop to foster mathematical understanding and expertise and skills and knowledge – what students need to know and be able to do. The mathematical content standards were built on progressions of topics across a number of grade levels, informed both by research on children’s cognitive development and by the logical structure of mathematics.”

Adapted from California Common Core State Standards – Mathematics  
Introduction, page 2

In August of 2014, Davis Joint Unified went through a process with all of the secondary math teachers to determine the pathways for implementation of the Common Core State Standards. Through this process it was unanimously decided to follow the Integrated Pathway to help our students become college and career ready along with having a cohesive TK-12 math program for all of our students.

The Integrated Pathway is made up of three courses (Mathematics I, II, and III). The integrated mathematics courses follow the structure began in the K-8 standards of presenting mathematics as a multifaceted, coherent subject, and is the way most other high performing countries present higher mathematics. Each course is comprised of standards selected from the six high school conceptual categories, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. They have defined, evaluated, and compared functions, and used them to model relationships between quantities. Students have worked with radicals and applied the laws of exponents to situations involving integer exponents.

**Context for Course:** List the State/District Standards addressed in this course.  
See attached Standards Sheet.

**History of Course Development:** See Background Information.

## COURSE GOALS AND/OR MAJOR STUDENT OUTCOMES

Description of how this course supports district goal to increase student awareness and appreciation of diversity:

## COURSE OBJECTIVES

The focus of the Mathematics II course is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Some standards are repeated in multiple higher mathematics courses; therefore instructional notes, which appear in brackets, indicate what is appropriate for study in this particular course. For example, the scope of Mathematics II is limited to quadratic expressions and functions, and some work with absolute value, step, and functions that are piecewise-defined. Therefore, although a standard may include references to logarithms or trigonometry, those functions should not be included in course work for Mathematics II; they will be addressed in Mathematics III.

For the Mathematics II course, instructional time will focus on five critical areas: (1) extend the laws of exponents to rational exponents; (2) compare key characteristics of quadratic functions with those of linear and exponential functions; (3) create and solve equations and inequalities involving linear, exponential, and quadratic expressions; (4) extend work with probability; and (5) establish criteria for similarity of triangles based on dilations and proportional reasoning.

## COURSE OUTLINE

### Integrated Mathematics II Course Outline

#### Number and Quantity

##### The Real Number System

- ☐ Extend the properties of exponents to rational exponents.
- ☐ Use properties of rational and irrational numbers.

##### The Complex Number Systems

- ☐ Perform arithmetic operations with complex numbers.
- ☐ Use complex numbers in polynomial identities and equations.

#### Algebra

##### Seeing Structure in Expressions

- ☐ Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

##### Arithmetic with Polynomials and Rational Expressions

- ☐ Perform arithmetic operations on polynomials.

##### Creating Equations

- ☐ Create equations that describe numbers or relationships.

##### Reasoning with Equations and Inequalities

- ☐ Solve equations and inequalities in one variable.
- ☐ Solve systems of equations.

#### Functions

##### Interpreting Functions

- ☐ Interpret functions that arise in applications in terms of the content.
- ☐ Analyze functions using different representations.

##### Building Functions

- ☐ Build a function that models a relationship between two quantities.
- ☐ Build new functions from existing functions.

##### Linear, Quadratic, and Exponential Models

- ☐ Construct and compare linear, quadratic, and exponential models and solve problems.
- ☐ Interpret expressions for functions in terms of the situation they model.

**Trigonometric Functions**

- ☐ Prove and apply trigonometric identities.

**Geometry****Congruence**

- ☐ Prove geometric theorems.

**Similarity, Right Triangles, and Trigonometry**

- ☐ Understand similarity in terms of similarity transformations.  
☐ Prove theorems involving similarity.  
☐ Define trigonometric ratios and solve problems involving right triangles.

**Circles**

- ☐ Understand and apply theorems about circles.  
☐ Find arc lengths and areas of sectors of circles.

**Expressing Geometric Properties with Equations**

- ☐ Translate between the geometric description and the equation for a conic section.  
☐ Use coordinates to prove simple geometric theorems algebraically.

**Geometric Measurement and Dimension**

- ☐ Explain volume formulas and use them to solve problems.

**Statistics and Probability****Conditional Probability and the Rules of Probability**

- ☐ Understand independence and conditional probability and use them to interpret data.  
☐ Use the rules of probability to compute probabilities of compound events in a uniform probability model.

**Using Probability to Make Decisions**

- ☐ Use probability to evaluate outcomes of decisions.

**TEXTS AND SUPPLEMENTAL INSTRUCTIONAL MATERIALS**

Integrated Math II: *Core Connections, Integrated II*, CPM (2014)

Previously Adopted? ☐ Yes ☒ No (If no, provide information directly below)

Cost per book TBD

Total Cost TBD

Budget Source: Instructional  
Materials Budget

**DIFFERENTIATED INSTRUCTIONAL METHODS AND/OR STRATEGIES****Strategies for Supporting School Goal of Improving Writing Skills:**

Certain assignments have required written components, requiring original expression and articulating reasoning for problem solving.

**ASSESSMENT METHODS AND/OR TOOLS**

Pre and Post tests  
 Formative Assessments  
 Benchmark Assessments  
 Summative Assessments  
 Performance Tasks  
 Written work  
 Observation record of student performance  
 Completion of assignments and worksheets

### ASSESSMENT CRITERIA

Students are presented with course criteria and necessary skills and concepts. This is followed by formative assessments, benchmark assessments and summative assessments with diverse types of performance tasks. Learning is successive, providing students opportunities for mastery over the course of the year.

### HONORS COURSES ONLY

#### Sequence Participation

**Describe what sequence this course would be a part of, and what role in the sequence this course plays.**

This course is part of the continued sequence of Common Core Mathematics in the Integrated Pathway.

#### Post-Secondary Articulation

**Indicate what post-secondary articulation this sequence offers: is this course part of a pathway that results in certification or college credit?**

The sequence of courses is articulated with CSU and UC systems.

## Number and Quantity

### The Real Number System

**N-RN**

#### Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### The Complex Number System

**N-CN**

#### Perform arithmetic operations with complex numbers. [ $i^2$ as highest power of $i$ ]

1. Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
2. Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

#### Use complex numbers in polynomial identities and equations. [Quadratics with real coefficients]

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

## Algebra

### Seeing Structure in Expressions

**A-SSE**

#### Interpret the structure of expressions. [Quadratic and exponential]

1. Interpret expressions that represent a quantity in terms of its context. ★
  - a. Interpret parts of an expression, such as terms, factors, and coefficients. ★
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1 + r)^n$  as the product of  $P$  and a factor not depending on  $P$ .* ★
2. Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

Note: ★ Indicates a modeling standard linking mathematics to everyday life, work, and decision-making. (+) Indicates additional mathematics to prepare students for advanced courses.

**Write expressions in equivalent forms to solve problems.** [Quadratic and exponential]

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  - a. Factor a quadratic expression to reveal the zeros of the function it defines. ★
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★
  - c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* ★

**Arithmetic with Polynomials and Rational Expressions**

**A-APR**

**Perform arithmetic operations on polynomials.** [Polynomials that simplify to quadratics]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Creating Equations**

**A-CED**

**Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* CA ★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★ [Include formulas involving quadratic terms.]

**Reasoning with Equations and Inequalities**

**A-REI**

**Solve equations and inequalities in one variable.** [Quadratics with real coefficients]

4. Solve quadratic equations in one variable.
  - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.
  - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

**Solve systems of equations.** [Linear-quadratic systems]

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

## Functions

### Interpreting Functions

**F-IF****Interpret functions that arise in applications in terms of the context.** [Quadratic]

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

**Analyze functions using different representations.** [Linear, exponential, quadratic, absolute value, step, piecewise-defined]

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
  - a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  - b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ , and  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

### Building Functions

**F-BF****Build a function that models a relationship between two quantities.** [Quadratic and exponential]

1. Write a function that describes a relationship between two quantities. ★
  - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
  - b. Combine standard function types using arithmetic operations. ★

**Build new functions from existing functions.** [Quadratic, absolute value]

3. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*



4. Find inverse functions.

- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$ .

**Linear, Quadratic, and Exponential Models**

**F-LE**

**Construct and compare linear, quadratic, and exponential models and solve problems.** [Include quadratic.]

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

**Interpret expressions for functions in terms of the situation they model.**

6. Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity. CA ★

**Trigonometric Functions**

**F-TF**

**Prove and apply trigonometric identities.**

8. Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

**Geometry**

**Congruence**

**G-CO**

**Prove geometric theorems.** [Focus on validity of underlying reasoning while using variety of ways of writing proofs.]

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

**Similarity, Right Triangles, and Trigonometry**

**G-SRT**

**Understand similarity in terms of similarity transformations.**

1. Verify experimentally the properties of dilations given by a center and a scale factor:
- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

**Prove theorems involving similarity.** [Focus on validity of underlying reasoning while using variety of formats.]

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Define trigonometric ratios and solve problems involving right triangles.**

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
  7. Explain and use the relationship between the sine and cosine of complementary angles.
  8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★
- 8.1 Derive and use the trigonometric ratios for special right triangles ( $30^\circ$ ,  $60^\circ$ ,  $90^\circ$  and  $45^\circ$ ,  $45^\circ$ ,  $90^\circ$ ). CA

## Circles

G-C

**Understand and apply theorems about circles.**

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

**Find arc lengths and areas of sectors of circles.** [Radian introduced only as unit of measure]

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. **Convert between degrees and radians.** CA

## Expressing Geometric Properties with Equations

G-GPE

**Translate between the geometric description and the equation for a conic section.**

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.

**Use coordinates to prove simple geometric theorems algebraically.**

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ . [Include simple circle theorems.]*
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**Geometric Measurement and Dimension**

**G-GMD**

**Explain volume formulas and use them to solve problems.**

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★
5. Know that the effect of a scale factor  $k$  greater than zero on length, area, and volume is to multiply each by  $k$ ,  $k^2$ , and  $k^3$ , respectively; determine length, area and volume measures using scale factors. CA ★
6. Verify experimentally that in a triangle, angles opposite longer sides are larger, sides opposite larger angles are longer, and the sum of any two side lengths is greater than the remaining side length; apply these relationships to solve real-world and mathematical problems. CA

**Statistics and Probability**

**Conditional Probability and the Rules of Probability**

**S-CP**

**Understand independence and conditional probability and use them to interpret data.** [Link to data from simulations or experiments.]

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ★
2. Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
3. Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ . ★
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. ★

**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

6. Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model. ★
7. Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model. ★
8. (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model. ★
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. ★

**Using Probability to Make Decisions****S-MD****Use probability to evaluate outcomes of decisions.** [Introductory; apply counting rules.]

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). ★